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Symmetry breaking in the early universe and accelerated frames†

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Abstract. The formulation of symmetry breaking in a manifold with non-trivial metric is discussed. It is shown that a conformally coupled self-interacting scalar field exhibits spontaneous symmetry breaking in a RW universe. The non-trivial implication of this result to cosmology is discussed. The effective potential in the static space–time is computed to see the effect of one-loop connections. A self-consistent solution is given. In the later part of the paper it is suggested that accelerated frames also have similar effects on the symmetry breaking scheme. Its implications are discussed.

1. Introduction

One of the major triumphs of theoretical physics in the previous decade was the unification of electromagnetic and weak interactions (Abers and Lee 1973). The success of this theory depends crucially on the idea of spontaneous symmetry breaking (SSB).

It would be necessary sooner or later to include the effects of gravity into the scheme. To the extent that quantum gravity effects are negligible, this can be achieved by doing field theory in a curved space–time (for a review, see Gibbons (1979)). The gross features of gravity are already being incorporated in the applications of GUTS to the early universe scenario. But it is also necessary to examine the *structure* of the theory in a curved space–time. Qualitatively speaking, symmetry breaking arises because the asymmetric state is energetically favourable to the symmetric one. Hence, if there exists any external interaction (like the existence of the non-zero temperature), it can restore or break the symmetry, depending on the relative strength. Since gravitational interactions cannot be neglected during the early stages of the universe, it is necessary to examine this question in detail.

There is another motivation for this study. It has been noted by various people (see for example De Witt 1979) that a uniformly accelerated frame resembles, in some sense, a thermal bath. It is well known that the symmetry of a spontaneously broken theory can be restored at sufficiently high temperatures (Linde 1979). These results, when combined, raise serious questions about the general covariance of the scheme. This is a crucial point to be settled before one can meaningfully talk about the field theory in the curved space–time.

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The above is applicable to Schwarzschild and de Sitter space–times as well—since they also behave like 'space–times with a thermal bath' (Gibbons and Hawking 1977). Naively, one would expect the symmetry to be restored at small mass values of the Schwarzschild space–time.

We shall study some of these questions in this paper. We begin by considering a self-interacting ($\lambda\phi^4$) scalar field in an open Robertson–Walker space–time. We show that a conformal coupling to the curvature can induce symmetry breaking in a very natural way. (No assumptions need be made about the bare mass and coupling constant values.) The massive model is also discussed, which shows symmetry breaking during a particular phase of the evolution. This result has certain non-trivial implications for cosmology which are discussed in the next section. The need for an external source for the metric is dispelled in § 4 by using the vacuum energy density of the scalar field as the source.

All the above results are valid in the strict sense only at tree level. The conformal coupling is also crucially used. In § 5, we use the technique of effective potential to incorporate the quantum closed loop corrections. Here we do away with the conformal coupling and consider symmetry breaking to arise from radiative corrections.

In §§ 6 and 7 we discuss some features of symmetry breaking in the non-inertial coordinates. It is shown that symmetry breaking can be induced in the Milne coordinates while no such effect arises in the standard accelerated Rindler coordinates. However, the Rindler coordinates do act as a thermal bath as far as restoring the symmetry is concerned. The implication of this analysis is unclear.

2. Scalar field in the RW universe

Consider a space–time described by the metric (units: $c = h = 1$)

$$ds^2 = a^2(t)[dt^2 - d\chi^2 - \sinh^2\chi(d\theta^2 + \sin^2\theta\, d\phi^2)]. \tag{2.1}$$

Here $a(t)$ is a given function to be determined by solving Einstein's equation. Our analysis is independent of this functional form. Consider a Lagrangian for a scalar field ϕ, of the form,

$$L = \tfrac{1}{2}(g^{\mu\nu}\, \partial_\mu\phi\, \partial_\nu\phi - \tfrac{1}{6}R\phi^2 - \tfrac{1}{6}\lambda\phi^4). \tag{2.2}$$

Here R is the scalar curvature corresponding to the metric in equation (2.1) and this makes the interaction conformally invariant. The term $\frac{1}{6}\lambda\phi^4$ (with $\lambda > 0$) is added to make the theory renormalisable. In any interaction this term will, anyway, arise as a radiative correction. In this section, however, we will not worry about quantum corrections and will work at tree level. Also notice that the bare mass term is absent in the Lagrangian. The field equation reads

$$\nabla_\mu\nabla^\mu\phi + \tfrac{1}{6}R\phi + \tfrac{1}{3}\lambda\phi^3 = 0. \tag{2.3}$$

Consider now the vacuum expectation value (VEV) of the field $\phi(x)$. Let

$$\langle 0|\phi(\bar{x}, t)|0\rangle = \eta(\bar{x}, t). \tag{2.4}$$

In standard field theory, the Lagrangian is translationally invariant, and hence η cannot depend on the space–time coordinates. In field theory with an external potential, the Lagrangian possesses only a limited symmetry. We can impose on η only those symmetries respected by the Lagrangian. Thus, from the isotropy and homogeneity of the universe, it follows that

$$\eta(t, \bar{x}) = \eta(t). \tag{2.5}$$

One has now to decide the form of $\eta(t)$. In principle this can be done only through calculating the effective (quantum) potential. However, at tree level, this can be done very simply by taking the VEV of equation (2.3) and using, at tree level,

$$\langle 0|\phi^3|0\rangle = \langle 0|\phi|0\rangle^3 = \eta^3. \tag{2.6}$$

This leads to the equation

$$\nabla_\mu \nabla^\mu \eta + \tfrac{1}{6}R\eta + \tfrac{1}{3}\lambda\eta^3 = 0 \tag{2.7}$$

or, in expanded form,

$$\ddot{\eta} + (2\dot{a}/a)\dot{\eta} + (\ddot{a}/a - 1)\eta + \tfrac{1}{3}\lambda a^2\eta^3 = 0 \tag{2.8}$$

(where dot denotes differentiation WRT time). Quite clearly $\eta = 0$ is a solution where there is no symmetry breaking. The question is to find out whether that is the most stable solution. Make a substitution (suggested by the fact that the equations are conformally invariant)

$$\eta(t) = (3/\lambda)^{1/2} f(t)/a(t) \tag{2.9}$$

in equation (2.8), which will lead to

$$\ddot{f} + f^3 - f = 0. \tag{2.10}$$

This is the Duffin–Loret equation well known in the theory of oscillations. By inspecting its general solution (which is an elliptic integral)

$$t = \int \mathrm{d}f (c + f^2 - \tfrac{1}{2}f^4)^{-1/2} \qquad -\tfrac{1}{2} \leqslant c < \infty \tag{2.11}$$

or by considering the behaviour of small perturbations, one can easily see that the stable solution is *not* $f = 0$ *but* $f = \pm 1$. Thus the stable solution has a non-zero VEV for $\phi(x)$ given by

$$\langle 0|\phi(t, x)|0\rangle = \eta(t) = \pm(3/\lambda)^{1/2}(1/a(t)). \tag{2.12}$$

In a later section, we shall explicitly show that this solution is energetically more favourable than the $\eta = 0$ solution.

Notice that this symmetry breaking arises solely because of the existence of the non-trivial metric. One need not assume any special values for the parameters that appear in the Lagrangian. The physical field can be obtained from the $\phi(x)$ by the usual shift,

$$\psi(x) = \phi(x) - \eta(x) \tag{2.13}$$

which will lead to a time dependent mass for $\psi(x)$ given by

$$m_\psi^2(t) = 3/a^2(t) \tag{2.14}$$

which vanishes for large expansion factor.

Now let us consider the effect of introducing a bare mass into the Lagrangian (2.2). Consider now the Lagrangian, (2.2)

$$\mathscr{L}=\tfrac{1}{2}[g^{\mu\nu}\,\partial_\mu\phi\,\partial_\nu\phi-(m^2+\tfrac{1}{6}R)\phi^2-\tfrac{1}{6}\lambda\phi^4]. \tag{2.15}$$

This leads to the following equation for $f(t)$

$$\ddot{f}+(m^2a^2(t)-1)f+f^3=0. \tag{2.16}$$

Unfortunately, one cannot solve this equation in closed form for an arbitrary $a(t)$. However, the following conclusions can be made. In the early stages of the universe, $a(t)\sim 0$ and hence our previous analysis is applicable. There is a symmetry breaking at this epoch. However, as $a(t)$ increases, the coefficient of f will become large and positive. As one can see, by an analysis of small perturbations, this would make the $f=0$ solution stable. Thus only for a particular epoch would the symmetry remain broken.

3. Phase transitions and *CP* violation

The above analysis has non-trivial implications in a realistic field theory. First of all, consider the question of phase transitions in finite-temperature field theory. In the standard GUTS model, symmetry is expected to be restored during the early stages of the universe. This arises partly because of the high-temperature symmetry restoration. However, our analysis shows that a symmetry-breaking effect can arise because of the same metric. From equation (2.12) it is clear that this effect tries to restore symmetry at large times ($a\to\infty$) and tries to break the symmetry at the early stages of the universe ($a\to 0$). Thus this opposes the normal high-temperature behaviour and hence it is crucial to decide which effect predominates. This can be done as follows.

One can roughly estimate the temperature required for symmetry restoration (T_{SR}) as

$$kT_{\mathrm{SR}}\gtrsim\langle 0|\phi|0\rangle c^2=\eta(t)c^2 \qquad (k,c\neq 1) \tag{3.1}$$

so that

$$T_{\mathrm{SR}}\approx(3/\lambda)(1/a(t)) \qquad (\text{units: } k=c=h=1). \tag{3.2}$$

But, as is well known, the temperature of the early universe (Weinberg 1972) falls as

$$T_{\text{universe}}=B/a(t) \qquad B=\text{constant}. \tag{3.3}$$

In other words,

$$T_{\mathrm{SR}}/T_{\text{universe}}=\text{constant, independent of time}. \tag{3.4}$$

Thus the actual temperature of the universe is always higher or always lower than the temperature required for the symmetry restoration. In other words, no high-temperature phase transitions can occur in this simple model!

One should keep in mind that the model is very idealised. Still it shows the necessity of taking into account detailed features of gravitational interactions before confident predictions about the early universe can be made.

Another important feature arises when one considers a realistic system of fields. Notice that we have introduced here a Higgs scalar field and have broken its symmetry in a very natural way. If one considers vector or spinor fields interacting with $\phi(x)$, these matter fields will pick up masses which are time dependent.

In particular one can consider a coupling to a spinor field of the form

$$\mathcal{L}_I = g\bar{\psi}\gamma_5\psi\phi$$

and treat $\phi(x)$ as a pseudoscalar. When the symmetry is broken this will lead to a P violation and, since C is maintained, to a CP violation as well. Thus a model universe built on these lines has an intrinsic CP violating mechanism.

4. Self-consistent solution

In the previous sections $a(t)$ was taken to be an arbitrarily assigned function. The question arises as to the source for this non-trivial metric. While one can always assume existence of background matter compared to which the effect of the scalar field is negligible, it is of importance to see whether the scalar field itself can function as its source.

It has become a reasonably standard practice to use the vacuum expectation value of the $T_{\mu\nu}$ as the source for the geometry. In the present case, since ϕ itself has a VEV, no divergence difficulties will arise in the lowest order. The energy momentum tensor for our Lagrangian reads as

$$T_{\mu\nu} = \partial_\mu\phi\ \partial_\nu\phi - (1/\sqrt{-g})g_{\mu\nu} - \tfrac{1}{3}(R_{\mu\nu} + \nabla_\mu\nabla_\nu - g_{\mu\nu})\phi^2. \tag{4.1}$$

A straightforward calculation leads to the result for the $m = 0$ case, as

$$\varepsilon(t) = \langle 0|T^0_0|0\rangle = -3/2\lambda a^4(t) \tag{4.2}$$

$$p(t) = -\langle 0|T^i_i|0\rangle = -1/2\lambda a^4(t). \tag{4.3}$$

Now one has to solve the coupled Einstein scalar equation written as

$$R^\mu_\nu - \tfrac{1}{2}\delta^\mu_\nu R = -8\pi G\langle 0|T^\mu_\nu|0\rangle. \tag{4.4}$$

When expanded this reads explicitly as

$$\dot{a}^2 - 2a\ddot{a} + a^2 = -4\pi G/3\lambda; \qquad a^2 - \dot{a}^2 = 4\pi G/3\lambda. \tag{4.5}$$

These are solved by the simple function

$$a(t) = (4\pi G/3\lambda)^{1/2}\cosh t. \tag{4.6}$$

In other words the self-consistent solution with the symmetry breaking leads to a non-singular evolution for the geometry. (Notice that $a \neq 0$ for $t = 0$.) This, of course, is possible because of equation (4.2), giving a negative energy density, thus violating the condition on the singularity theorem.

In some sense we are doing a back reaction analysis here. In general, however, other matter besides the scalar field would be present. Because of the nonlinear nature of Einstein's equations it is not easy to find a self-consistent solution. However, one general remark can be made. If one considers the total Lagrangian for the system (there will be an explicit term representing gravity) it reads as

$$\mathcal{L} = (R/2k + \tfrac{1}{2}g^{\mu\nu}\ \partial_\mu\phi_\nu\partial\ \phi - R\phi^2/12 - \lambda\phi^4/12) \tag{4.7}$$

$$= (1/2k - \tfrac{1}{12}\phi^2)R + (\tfrac{1}{2}g^{\mu\nu}\ \partial_\mu\phi_\nu\partial_\nu\phi - \lambda\phi^4/12) \qquad (k = 8\pi G). \tag{4.8}$$

If $\langle 0|\phi^2|0\rangle \neq 0$, one can consider, to tree level, $k_{\text{eff}}^{-1} = k^{-1} - \frac{1}{6}\langle\phi\rangle^2$ as the effective gravitational constant. Note that this effective gravitational constant can be negative for sufficiently large $\langle\phi^2\rangle$ values, making gravitation repulsive. This, in a qualitative sense, explains the non-singular solution (4.6).

5. Quantum corrections and effective potentials

In our analysis of the previous models we have confined our attention to the tree-level result. The potential term of the Lagrangian can be used only at this tree level to find the classical minima (and to decide on symmetry breaking).

When quantum corrections are taken into account, the bare potential is replaced by an effective potential which can be computed as a series in closed loops. However, like any other field-theory correction, this involves manipulation of divergent quantities, and is meaningful only for renormalisable theories. One major feature of this approach is the possibility of symmetry breaking due to radiative (closed loop) corrections, first considered in Coleman and Weinberg (1973). This arises when the minimum of the effective potential lies at a non-zero value of the field.

We shall present the effective potential for a scalar field at single-loop order. The rationale and methods of evaluation of the effective potential are discussed in various references (see, for example, Coleman (1973)) and will be omitted (we shall also omit straightforward but long algebra!).

Since the aim is to consider symmetry breaking as an effect of radiative corrections, we shall omit the explicit curvature coupling used in the previous sections. Consider the Lagrangian (bare Lagrangian; counter terms for renormalisation omitted)

$$\mathscr{L} = \sqrt{-g}\left(\tfrac{1}{2}g^{\mu\nu}\,\partial_\mu\phi\,\partial_\nu\phi - \tfrac{1}{2}m^2\phi^2 - \tfrac{1}{12}\lambda\phi^4\right) \tag{5.1}$$

in the metric given by

$$ds^2 = dt^2 - a^2[d\chi^2 + \sinh^2\chi(d\theta^2 + \sin^2\theta\, d\phi^2)]. \tag{5.2}$$

In the spirit of the previous section one should have assumed $a = a(t)$ arbitrary. However, mathematical complications are enormous when this is attempted. In order to simplify the situation, we shall make the metric static, by assuming $a(t) = a =$ constant. (Our analysis in § 2 can still be applied for $a(t) = a =$ constant. Equation (2.8) reduces to

$$\ddot{\eta} - \eta + \tfrac{1}{3}\lambda a^2\eta^3 = 0 \tag{5.3}$$

which has the same structure as the equation (2.10).)

To the lowest order (at tree level) the potential is given by

$$V_0(\phi_c) = \tfrac{1}{2}m^2\phi_c^2 + \tfrac{1}{12}\lambda\phi_c^4 \tag{5.4}$$

where $\phi_c = \langle 0|\phi|0\rangle$ is the 'classical' potential. The single-loop correction can be evaluated by the method of Weinberg (Coleman and Weinberg 1973). One expands the field ϕ about ϕ_c in the Lagrangian and retains terms up to quadratic (regarding ϕ_c as a parameter). Let

$$\phi = \phi_c + \eta; \tag{5.5}$$

then

$$L_{\text{eff}} = \sqrt{-g}\left(\tfrac{1}{2}g^{\mu\nu}\,\partial_\mu\eta\,\partial_\nu\eta - \tfrac{1}{2}m^2\eta^2 - \tfrac{1}{4}\lambda\phi_c^2\eta^2\right). \tag{5.6}$$

The effective Hamiltonian for this Lagrangian is

$$H_{\text{eff}}(\phi_c, \eta) = \int d^3\bar{x}\sqrt{-\bar{g}}[\tfrac{1}{2}(\partial_0\eta)^2 + \tfrac{1}{2}g^{ij}_{(3)}\,\partial_i\eta\,\partial_j\eta + \tfrac{1}{2}m^2\eta^2 + \tfrac{1}{4}\lambda\phi_c^2\eta^2]. \tag{5.7}$$

The theory with H_{eff} as the Hamiltonian has a vacuum state $|0, \phi_c\rangle$ which depends on ϕ_c as a parameter. The effective potential is given (at unit proper volume) by the expression (see Coleman 1973)

$$V_1(\phi_c) = \langle 0|H(\phi_c, \eta)|0\rangle. \tag{5.8}$$

This evaluation is straightforward, but tedious. One has to decompose η into creation and annihilation operators. The wave equation for η reduces to the eigenvalue problem with the static Hamiltonian in equation (5.7). Fortunately this case is well analysed in literature (see Grib *et al* 1976). One can write

$$\eta(\bar{x}, t) = \int_0^\infty \frac{d\alpha}{((2\omega(\alpha))^{1/2}} \sum_{l,k} [a_{\alpha lk}\Phi_{\alpha lk}\exp(-i\omega(\alpha)t) + \text{HC}] \tag{5.9}$$

where $\Phi_{\alpha lk}(x)$ satisfies the eigenvalue equation

$$\left(-\frac{1}{\sqrt{-g_{(3)}}}\frac{\partial}{\partial x^i}\left(\sqrt{-g_{(3)}}g^{ij}\frac{\partial}{\partial x^j}\right) + m^2 + \frac{\lambda}{2}\phi_c^2\right)\Phi_{\alpha lk} = \omega^2(\alpha)\Phi_{\alpha lk} \tag{5.10}$$

with $\omega^2(\alpha) = (1/a^2)(\alpha^2+1) + m^2 + \frac{1}{2}\lambda\phi_c^2$ and $a_{\alpha lk}$ denoting the creation operator for the particular mode. The detailed form of $\Phi_{\alpha lk}$ is complicated and is not required by us. The only result we need (which is proved in Grib *et al* 1976) is

$$\sum_{l,k} |\Phi_{\alpha lk}|^2 = \alpha^2/2\pi^2a^3. \tag{5.11}$$

Substituting in equation (5.8), the effective potential (per unit proper volume) turns out to be as follows.

$$\begin{aligned}\langle 0, \phi_c|H_{\text{eff}}|0, \phi_c\rangle &= \tfrac{1}{2}\int \sqrt{-g_{(3)}}\,d^3x \int_0^\infty d\alpha\,\omega(\alpha)\sum_l |\Phi_{\alpha lk}|^2 \\ &= \tfrac{1}{2}\int \sqrt{-g_{(3)}}\,d^3x \int_0^\infty d\alpha\,\omega(\alpha)\frac{\alpha^2}{2\pi^2a^3}.\end{aligned} \tag{5.12}$$

$$V_1(\phi_c) = \frac{1}{4\pi^2a^3}\int_0^\infty dx\,x^2\omega(x). \tag{5.13}$$

As usual any straightforward evaluation of this integral will lead to a divergent result. This is to be expected since this involves the closed-loop corrections. However, one can renormalise the theory as regards the mass term ($\sim\phi^2$), interaction term ($\lambda\phi^4$) as well as a conformal coupling factor ($\sim\phi^2/a^2$). The last correction is similar to mass renormalisation in a theory with zero bare mass. Keeping these in mind, one can evaluate the integral in (5.13) with a series of cut-offs as ($M^2 = m^2 + (1/a^2) + (\lambda/2)\phi_c^2$)

$$V_1(\phi_c) = (1/64\pi^2)[M^4\ln(M^2/P^2) + z_1\phi_c^2 + z_2\phi_c^4 + z_3(\phi_c^2/a^2)]. \tag{5.14}$$

The finite parts of the renormalisation constants can be specified by the usual techniques (see Coleman and Weinberg 1973). The momentum cut-off P has to be taken of the order of characteristic mass of the theory. Performing these calculations leads to the following final results for three different bare Lagrangians.

5.1. Massive case $(m \neq 0)$ $m^2 > 0$

The symmetry breaking is hinted at by the existence of a non-zero minimum. In other words $U(\phi_c) = \partial(V_0 + V_1)/\partial\phi_c^2 = 0$ for a $\phi_c \neq 0$. In this case,

$$U(\phi_c) = m^2 + \tfrac{1}{6}\lambda\phi_c^2 + (\lambda/64\pi^2)[2M^2 \ln(m^2/M^2) - \lambda\phi_c^2 - (2/a^2)] \tag{5.15}$$

which is everywhere positive. Hence no symmetry breaking occurs.

5.2. Massive case $(m \neq 0)$ *with* $m^2 < 0$

Here the symmetry is broken at the tree level with the VEV given by $(-6m^2)^{1/2}/\lambda$. The derivative of the effective potential goes as

$$U(\phi_c) = m^2 + \tfrac{1}{6}\lambda\phi_c^2 + (\lambda/32\pi^2)M^2[\ln(M^2/-2m^2) - 1]. \tag{5.16}$$

Since U increases ϕ_c, the symmetry is broken if and only if $u(0) < 0$ or

$$m^2 + (\lambda/32\pi^2 a^2)[\ln(1/-2m^2a^2) - 1] < 0 \tag{5.17}$$

or $a^2 > -(1/m^2)(1/x)$ (symmetry is restored otherwise) where x is the solution to

$$x(\ln \tfrac{1}{2}x - 1) = 32\pi^2/\lambda. \tag{5.18}$$

In other words symmetry which is originally broken is restored for small a^2 values—large curvatures.

5.3. Massless case $(m^2 = 0)$

Here the perturbation technique cannot be used, since infrared singularities exist. One has to do the regularisation at an arbitrary point which is of limited validity.

Thus we have noticed quite a few non-trivial features when radiative corrections are computed in this simplest space–time. The major one is the restoration of symmetry for large curvatures, starting with a theory with broken symmetry.

6. Symmetry breaking in a non-inertial frame Milne Universe

We shall consider an entirely different aspect of the symmetry breaking in the rest of the sections. It must be clear from § 2 that non-zero VEV's arise because of the non-trivial coefficients in the wave equation. Since such a situation can be mimicked to a great extent by the choice of a non-inertial frame, it is of interest to see how the broken symmetry theories behave in a non-inertial frame.

To do this in the simplest possible way, consider again the scalar field with $\lambda\phi^4$ interaction. Let the space be flat. In standard spherical coordinates we have

$$ds^2 = dT^2 - dr^2 - r^2(d\theta^2 + \sin^2\theta\, d\phi^2). \tag{6.1}$$

The VEV satisfies the equation

$$(\partial^2/\partial T^2 - \nabla^2)\eta + \lambda\eta^3 = 0. \tag{6.2}$$

Clearly, the stable solution is $\eta = 0$. Now consider the same flat space described in the Milne coordinates, given by

$$r = A\, e^t \sinh\chi \qquad T = Ae^t \cosh\chi \tag{6.3}$$

(A is introduced for the sake of dimensions) which leads to the metric

$$ds^2 = A^2 e^{2t}[dt^2 - d\chi^2 - \sinh^2\chi(d\theta^2 + \sin^2\theta\, d\phi^2)]. \tag{6.4}$$

This has exactly the same form as the RW metric considered in § 2. All the analysis in that section can be carried through leading to a VEV given by

$$\eta(t) = (1/\sqrt{\lambda})(e^{-t}/A). \tag{6.5}$$

Thus the symmetry breaking of the theory seems to depend crucially on the system of coordinates used! Further investigation is required to decide how physical this result is. (One should bear in mind that the transformation in (6.3) covers only a region of the space–time and hence is somewhat unphysical.) The result certainly cannot be accepted *per se* since it predicts that a symmetric theory can break the symmetry by a mere coordinate relabelling.

7. Rindler Universe

The coordinate transformation in (6.3) does not have a simple physical meaning. A very similar transformation given by

$$\begin{aligned}(1+gX) &= (1+gx)\cosh gt\\ gT &= (1+gx)\sinh gt\end{aligned} \tag{7.1}$$

takes the flat space metric

$$ds^2 = dT^2 - dX^2 - dy^2 - dz^2 \tag{7.2}$$

to the form

$$ds^2 = (1+gx)^2\, dt^2 - dx^2 - dy^2 - dz^2. \tag{7.3}$$

These are the coordinates suited to the uniformly accelerated observer—widely discussed in the literature (see Gibbons 1979). Our interest is to investigate a $\lambda\phi^4$ theory in this coordinate frame to see whether any new effects emerge. Let the Lagrangian be taken to be

$$L = \tfrac{1}{2}\sqrt{-g}(g^{\mu\nu}\,\partial_\mu\phi_\nu\,\partial\phi - \tfrac{1}{4}\lambda\phi^4). \tag{7.4}$$

We would like to check first whether this will lead to a symmetry breaking because of the existence of the non-trivial metric (7.3). Consider the VEV

$$\langle 0|\phi(x^\mu)|0\rangle = \eta(x^\mu). \tag{7.5}$$

Since $g_{\mu\nu}$ depends only on x, $\eta(x^\mu) = \eta(x)$ only. Following the, by now familiar, argument, one can write the equation satisfied by η as

$$d^2\eta/d\xi^2 - \tfrac{1}{3}\lambda\, e^{2g\xi}\eta^3 = 0 \tag{7.6}$$

where $g\xi = \ln|1+gx|$. By making a substitution

$$\eta(\xi) = e^{-g\xi}f(\xi) \tag{7.7}$$

one can reduce this to the form

$$\ddot{f} - g\dot{f} + g^2 f - \tfrac{1}{3}\lambda f^3 = 0 \qquad \text{(the dot denotes } \partial/\partial\xi). \tag{7.8}$$

By doing small perturbations about the solution one can see that $f = 0$ *is* indeed a stable solution. In other words the symmetry remains intact in this frame.

However, this Rindler system can induce another effect. Consider now a Lagrangian with a negative square term in the mass, so that the symmetry is spontaneously broken in the Minkowski coordinates. The symmetry can be restored in the accelerated frame (essentially it acts as a thermal bath). The derivation goes exactly in the same way as for any finite temperature result (Linde 1979). The Lagrangian is

$$L = (\tfrac{1}{2}g^{\mu\nu}\,\partial_\mu\phi\,\partial_\nu\phi + \tfrac{1}{2}\mu^2\phi^2 - \tfrac{1}{4}\lambda\phi^4)\sqrt{-g} \tag{7.9}$$

with the field equation

$$(\Box + \mu^2 - \lambda\phi^2)\phi = 0. \tag{7.10}$$

Let $\phi = \psi + \eta$ with $\langle 0|\psi|0\rangle = 0$, $\langle 0|\phi|0\rangle = \eta$. Taking the vacuum expectation value of equation (7.10) one gets

$$\Box\eta - (\lambda\eta^2 - \mu^2)\eta - 3\lambda\eta\langle\psi^2\rangle - \lambda\langle\psi^3\rangle = 0. \tag{7.11}$$

To the lowest order one can neglect the $\langle\psi^3\rangle\lambda$ term. (See Linde (1979) for a detailed discussion.) One has to quantise the physical field ψ in the accelerated frame introducing appropriate creation and annihilation operators. Then one gets

$$\langle 0|\psi^2|0\rangle = \frac{1}{(2\pi)^3}\int \frac{d^3p}{2\omega_p}\, 2\,\langle 0|a_p^+ a_p|0\rangle + \text{zero point term}. \tag{7.12}$$

In the accelerated frame, however, $\langle 0|a_p^+ a_p|0\rangle$ is not zero, but a Planckian distribution with the temperature $T = g/2$. So it gives

$$\langle 0|\psi^2|0\rangle = \frac{1}{2\pi^2}\int_0^\infty \frac{p^2\,dp}{P(e^{P/T} - 1)} = T^2/12 = g^2/48\pi^2. \tag{7.13}$$

Substituting into equation (7.11) one sees that there are two constant solutions,

$$\eta = 0 \qquad \text{and} \qquad \eta^2 = \mu^2/\lambda - T^2/4 = \mu^2/\lambda - g^2/16\pi^2. \tag{7.14}$$

Thus the symmetry is broken at low accelerations while it gets restored above values

$$g = 4\pi\mu/\sqrt{\lambda} \equiv (4\pi/\sqrt{\lambda})(\mu c^3/h). \tag{7.15}$$

(For a particle of electron mass this gives $10^{29}\,\text{cm s}^{-2}$.) In this respect, the general covariance of the spontaneously broken theory is again under suspicion.

8. Conclusions

We have shown that the inclusion of a non-Lorentzian metric (whether due to curvature or due to acceleration) changes the standard symmetry breaking results drastically. It is necessary to examine in detail how the predictions of a gauge theory, based on SSB, change when strong gravitational fields are present.

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